WATER ENTRY OF SHIP SECTIONS AND AXISYMMETRIC BODIES

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SUMMARY

Theoretical methods for water entry of two-dimensional and axisymmetric bodies are presented. When the local angle between the water surface and the body surface is very small, hydroelasticity must be considered. The presented slamming analysis assumes the structure is rigid. Two 2-D numerical methods are discussed. One of the methods simplifies the dynamic free surface condition and details of the jet flow. Flow separation from sharp corners are incorporated. The simplified method is generalized to 3-D axisymmetric flow without flow separation. In order to verify the 3-D method, water entry of axisymmetric bodies with small local deadrise angles are studied analytically by means of matched asymptotic expansions. A composite solution for the pressure is presented. The numerical method is verified by comparing with the asymptotic method and validated by comparing with experiments for cones and spheres. It is demonstrated that satisfaction of exact body boundary condition is more important than satisfaction of exact free surface conditions. The effect of local rise up of the water is significant.

1. INTRODUCTION

Impact between the water and a ship, i.e. slamming, can cause important local and global loads on a vessel. Different physical effects may have an influence. When the local angle between the water surface and the body surface is very small at the impact position, an air cushion may be formed between the body surface and the water surface. Compressibility of the air influences the air flow. The air flow interacts with the water flow, which is influenced by the compressibility of the water. When the air cushion collapses, air bubbles are formed. The large loads that can occur during impact between a nearly horizontal body and a water surface, can cause important dynamic hydroelastic effects. This can lead to subsequent cavitation and ventilation. These physical effects have different time scales. The important time scale from a structural point of view is when maximum stresses occur. This scale is the highest wet natural period for the local structure. The effect of compressibility and the formation and collapse of an air cushion are significant initially and normally in a time scale much smaller than the time scale of when maximum stresses occur. This may occur for wetdeck slamming. By wetdeck is meant the structural part connecting the side hulls of a multihull vessel. Compressibility and air cushion formation will then have a smaller effect on maximum local stresses relative to dynamic hydroelastic effects. The largest impact pressures occur initially and will have a minor effect on the maximum stresses. It is an initial force impulse that matters. The reasons are that the largest pressures have a short duration relative to highest natural period for the local structure and have a small spacial extent at a given time instant. Theoretical and experimental studies of wave impact on horizontal elastic plates of steel are presented in refs. [1], [2], [3] and [4]. Ref. [5] studied also aluminium plates. The effect on plates with a small angle is discussed in refs. [3] and [6]. It is demonstrated that dynamic hydroelastic effects are significant. Ref. [7] studied slamming against elastic wedges penetrating an initially calm water surface.

When the local angle between the water surface and the body surface is not small at the impact position, local hydroelastic effects are not important for slamming on ship cross-sections. The slamming pressures can then be used in a static structural response analysis to find local slamming induced stresses. The air flow is unimportant and the water can be assumed incompressible and the flow irrotational. Since water impact is associated with high fluid accelerations, gravity does not matter. The local rise-up of the water has a significant effect. The spray by itself is not important for slamming. The pressure inside the spray is close to atmospheric. But the generation of spray is associated with high pressure gradients on the hull. This is more dominant, the smaller the local angle between the water surface and the body surface is at the impact position. The water entry loads on a ship cross-section with bowflare will introduce global hydroelastic effects (whipping) of the ship. Flow separation from knuckles should then be accounted for. Two numerical methods for water entry of ship cross-sections will be reported. They have been validated by comparisons with model tests. One of the methods is exact within potential theory. The other represents a simplification and is more robust for engineering use. The theoretical and experimental studies show that common engineering methods to predict water entry loads on ship cross sections give too low maximum force and wrong time history of the force.

Generalizations of the two-dimensional methods to three-dimensional flow are needed. An asymptotic theory for water entry of an axisymmetric body with small local deadrise angles is presented. This represents a valuable tool for verification of three-dimensional methods. It is demonstrated that local rise up of the water at the impacting body is also significant for three-dimensional flow.

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The simplified numerical method derived for 2-D flow is generalized to water entry of axisymmetric bodies. The method is verified by comparing with the asymptotic method and validated by comparing with experiments for spheres and cones. It is demonstrated that satisfaction of exact body boundary conditions is more important than satisfaction of exact free surface conditions. It is possible to further develop the method and study water entry of a general 3-D structure.

2. SLAMMING LOADS ON SHIP CROSS-SECTIONS

Slamming on ship hulls is often categorized as bottom slamming and bow flare slamming. The physics of bottom slamming has similarities with wetdeck slamming. When a bow flare section of a ship enters the water, the local loads around the flare are not significantly influenced by hydroelasticity. The pressure distribution can for instance be estimated by the nonlinear boundary element method developed by Zhao and Faltinsen [8]. This method accounts for the local rise-up of the water and the spray generation during entry. The fine details of the spray are neglected, but this is believed to be unimportant for slamming pressures and integrated water entry loads. A reason is that the pressure is close to atmospheric pressure in the spray. Gravity is neglected since fluid accelerations are initially dominating relative to gravitational acceleration. Gravity may play a role at a later stage of the water entry. However, it is a priori believed that introduction of gravity in the numerical model will not cause any problems. The numerical method solves the two-dimensional Laplace equation for the velocity potential as an initial value problem. The exact free surface conditions without gravity and the exact body boundary condition are satisfied at each time instant. An important feature is how the intersection between the water surface and the body surface is handled. Since the fine details of the spray are not studied and the pressure can be approximated as atmospheric in the spray, the spray is excluded in the boundary element formulation. A control surface normal to the body surface is drawn through the spray root. This surface can be handled in a similar way as a free surface. The advantage of doing this is that the intersection between the water and the body at the free surface can be handled in a more robust way. Following the details of the jet flow associated with the spray could cause a small intersection angle between the free surface and the body surface. Small numerical errors would cause large errors in the prediction of the intersection points. This can then destroy the numerical solution. This is a more severe problem the more blunt the body is. Since the free surface will have a high curvature close to the body surface during water entry of a blunt body, the free surface shape was described in ref. [8] by a higher order description. If straight line elements are used, artificial mass may be generated and destroy the accuracy.

Very good agreement with similarity solutions for wedges with deadrise angles between 4^o and 81^o and with asymptotic solutions for small deadrise angles were documented in ref. [8]. The water entry velocity is constant. There is a peak in the pressure distribution at the spray root close to the free surface when the deadrise angle is less than approximately 30^o. This pressure peak is what is often referred to as the slamming pressure. When the deadrise angle is less than 20^o the pressure distribution becomes more and more peaked and concentrated close to the spray root and sensitive to the deadrise angle. A consequence of this is that rolling could have an important effect on the slamming loads. Faltinsen [9] validated the method by comparing with drop tests of a bowflare section with a constant heel angle of 22.5^o.

The original method does not include flow separation from knuckles or convex surfaces. Zhao, Faltinsen and Aarsnes [10] have extended the original method to include separation from knuckles. A Kutta condition implying tangential velocity and continuity in the pressure at a separation point is satisfied. The pressure becomes more uniformly distributed in space after flow separation. The magnitude of the pressure is still significant. Predicting flow separation from convex surfaces have not been studied. This is a harder problem because the separation point is a priori unknown. An iteration procedure is needed. The flow separation cannot be determined by a viscous flow analysis. The duration of the water entry is too short to develop velocity profiles with zero shear stress at a point on the surface. The latter is the normal criterion for flow separation due to viscous effects. The situation is believed to be more similar to cavity flow past a blunt body like a circle in cross-flow.

Even if hydroelasticity does not affect the local loads during water entry of a ship cross section, it may play an important role in a global analysis. By considering the ship hull as an elastic beam, the integrated water-entry force on for instance a bow flare section causes transient hydroelastic response (whipping) of the ship. The commonly used methods to predict whipping due to water entry of ship cross-sections do not account for the local rise-up of the water. Ref. [10] demonstrated that local rise-up of the water is important for water entry forces on bow flare sections. Its importance will increase with increasing relative vertical velocities between a ship cross-section and the water. The smaller this relative velocity is, the more important is Froude-Kriloff and hydrostatic forces. By Froude-Kriloff forces is meant the pressure loads due to the incident waves only. Ref. [10] also showed that the hydrodynamic water entry force is not negligible when the flow has separated from the knuckles. The peak in the vertical force occurs when the spray root is at the knuckle.

If the hydrodynamic vertical water entry forces are expressed in terms of the time derivative of infinite frequency added mass as a function of submergence relative to undisturbed free surface, the force part after flow separation from the knuckles will be negligible. This is common to do. An approach like this will also give too low maximum force and a wrong time history of the force. The reason is that an important part of the force is associated with the rate of change with time of the wetted area. The local rise up of the water implies a larger rate of change of the wetted area.

At a late stage of the water entry, it is of interest to compare the numerical results of vertical force with theoretical drag coefficients for steady symmetric cavity flow past a blunt body. These values are a function of the cavitation number.

Knapp et al. [11] defines the cavitation number as $K=(p_0-p_B)/0.5\rho V^2$ for water entry. Here V is the velocity of the body, p_0 is pressure in undisturbed fluid at the depth of the nose of the entering body, p_B is the cavity pressure. The cavity pressure is the same as atmospheric pressure in our case. Further ρ is the mass density of water. According to ref. [11], C_D is 0.745 for two-dimensional symmetric steady supercavitating flow (K=0) past a wedge with interior angle 120° at the nose. C_D is defined as $C_D=F/(0.5\rho V^2 B)$. Here F is the total force and B is the maximum breadth of the section. This body shape was studied in ref. [10]. Since gravity is neglected in the numerical computations, the water entry force on the wedge should approach the results for supercavitating flow when the submergence goes to infinity and the drop velocity is constant. The results show that the unsteady force part reduces slowly after a rapid decrease just after flow separation from the knuckles. The results were plotted as a function of the inverse of the submergence of the wedge. This makes it easier to estimate asymptotic values when Vt goes to infinity. Here t is the time variable. The results indicated that the computed C_D-value for a wedge with deadrise angle 30^o approaches an asymptotic value close to 0.745 when the submergence goes to infinity. The numerical simulations should ideally have been continued for larger submergences, but numerical difficulties were encountered. The computations indicated that the cavity becomes infinitely long when the submergence goes to infinity. It is expected that the cavity will be finite if gravity is included. The deceleration of the section will also affect the solution.

The presented method does not solve secondary impact problem. An example on this is water entry of a cross-section with a sonar dome. The jet flow separating from the sonar dome can cause secondary impact on the hull. The same may occur due to flow separation from the bottom of a heeled structure entering the water.

A simplified solution for water entry of a two-dimensional body was presented by Zhao, Faltinsen and Aarsnes [10]. The solution is more numerically robust and faster than the original method in ref. [8]. The quality of the predictions is believed to be satisfactory for engineering applications. The method is a generalization of Wagner's method. Wagner [12] developed an asymptotic solution for water entry of two-dimensional bodies with small local deadrise angles. The flow was studied in two fluid domains. The inner flow domain contains a jet flow at the intersection between the body and the free surface. In the outer flow domain the body boundary condition and the dynamic free surface condition $\phi=0$ where transformed to a horizontal line. The kinematic free-surface condition was used to determine the intersection between the free surface and the body in the outer flow domain. Satisfaction of the kinematic free surface condition implies that the displaced fluid mass by the body is properly accounted for as rise up of the water. This is not true for a von Karman approach that does not account for the local rise up of the water.

In the generalization of Wagner's solution to larger local deadrise angles only the outer flow domain solution is analyzed. A main difference from the Wagner theory is that the exact body boundary condition is satisfied at each time instant. The wetted body surface is found by integrating in time the vertical velocities of fluid particles on the free surface and finding when the particles intersect the body surface. Wagner did also that, but he could use analytical solutions due to the simplified boundary conditions. The dynamic free-surface condition is the same as Wagner used. This is a simplification relative to the more complete method presented in ref. [8]. The pressure is calculated by the complete Bernoulli's equation without gravity. It has not been possible to find an inner flow solution near the spray roots that matches the outer flow solution for finite deadrise angles. This would have made it possible to exclude in a rational way the large negative pressures that occur at the intersection points in the outer flow solution. The procedure is simply to neglect the negative pressures. The theory was verified by comparing with the fully nonlinear solution and validated by comparisons with model tests. The simplified theory shows the importance to satisfy the exact body boundary condition and include the local water elevation at the hull. The exact dynamic free-surface condition is less important.

The simplified nonlinear analysis in ref. [10] was extended by Zhao, Faltinsen and Haslum [13] to include flow separation from fixed separation points. This is done by an iterative process. A shape of the separated free-surface is assumed and the kinematic free-surface condition is satisfied on the assumed free surface. The functional form of the separated free-surface is found by an analytical solution close to the separation point. The shape of the separated free surface is iterated until the difference between the pressure on the separated free-surface and atmospheric pressure is minimized. A Kutta condition is satisfied at the separation points. The solution was verified by comparing with the fully nonlinear solution in ref. [10] for water entry of wedges with knuckles. The method was also validated by comparing with experiments from drop tests of a wedge and a bow flare section with knuckles.

3. ASYMPTOTIC THEORY FOR WATER ENTRY OF AN AXISYMMETRIC BODY

In order to generalize the two-dimensional numerical methods to three-dimensional methods for water entry of ships or other structures, analytic solutions are important as verification tools. This is a motivation for studying an asymptotic theory for water entry of an axisymmetric body with small local deadrise angles.

The theory of Wagner [12] for water entry of a two-dimensional body will be generalized to an axisymmetric body. It will be shown that an axisymmetric outer flow solution matches Wagner's inner two-dimensional jet flow solution and that a composite solution for the pressure distribution on the body can be constructed. The matching is similar as Cointe [14] did in two dimensions. Chuang [15] and Shiffman and Spencer [16] studied also the outer flow solution during water entry of an axisymmetric body. Chuang's solution is approximate and leads to different results than ours.

The body shape and definition of coordinate system are shown in Fig. 1. The water entry velocity *V* is assumed constant. The instantaneous draft relative to calm water is *Vt* where *t* is the time variable. The vertical distance from the lowest point of the structure to the intersection between the free surface and the body shape is $\eta_b(r)$ in the outer flow description. The corresponding radial coordinate r is c(t). β is a local deadrise angle and assumed to be small in the fluid domain. The fluid is assumed incompressible and the flow irrotational. Boundary conditions in the outer flow are shown in Fig. 2. Since β is assumed small, the body boundary conditions are transferred to the lower part of a circular disc with radius c(t). The dynamic free surface condition $\phi=0$ is satisfied on a horizontal plane as shown in Fig. 2. The kinematic free surface condition is used to find c(t).

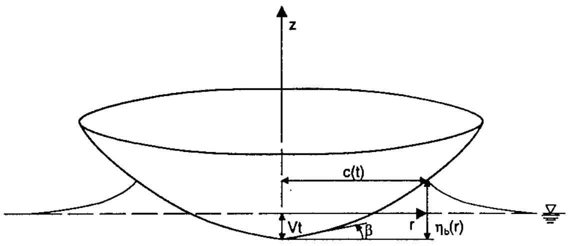

Fig. 1 Water entry of axisymmetric body. Definition of coordinate system.

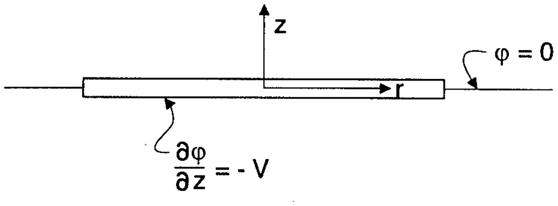

Fig. 2 Boundary conditions in the outer flow domain used in the analysis of water entry of a blunt axisymmetric body.

We need the velocity potential ϕ on $z=0^-$, r<c. We use the solution in Milne-Thomson [17] on p.499-501. The velocity potential is given by

$$\phi = -\frac{2V}{\pi}\sqrt{c^2-r^2} \qquad (1)$$

The solution agrees with Chuang's [15] solution. When we want to find the vertical velocity *w* at z=0 for r>c, it is convenient to start out with the stream function ψ. It follows that

$$w = \frac{2V}{\pi}\left\{\left((\frac{r}{c})^2-1\right)^{-1/2} - \sin^{-1}\left(\frac{c}{r}\right)\right\} \qquad (2)$$

When r/c → ∞, w ~ $2Vc^3/(3\pi r^3)$. The behavior is as expected like the flow due to a vertical dipole with singularity at r=0 and z=0. We should note that the dominant term when r → c agrees with ref. [15], but not the complete expression given by Eq. (2). When r → ∞, his solution for *w* behaves like r^{-2} and does not have the proper behavior.

We now want to find the intersection line between the body surface and the free surface like Wagner [12] did in the outer flow problem in two dimensions (see also Faltinsen [18]). The free surface elevation at the body and relative to the bottom of the structure can be found by integrating *w* in time and adding *Vt*. This has to be equal to $\eta_b(r)$ which is the vertical coordinate of a point on the structure relative to the bottom (see Fig. 1). We can write

$$\eta_b(r) = \int_0^r \left[\frac{2}{\pi}\left\{\left((\frac{r}{c})^2-1\right)^{-1/2} - \sin^{-1}\left(\frac{c}{r}\right)\right\} + 1\right]\mu(c)dc \qquad (3)$$

where

$$\mu(c) = Vdt/dc \qquad (4)$$

Eq. (3) is an integral equation for $\mu(c)$. When $\mu(c)$ is found, we use Eq. (4) to find c(t). We write $u(c) = \Sigma A_n c^n$. We will in practice be interested in n=0 and 1. It follows that Eq. (3) can be written

$$\eta_b(r) = A_0\frac{4}{\pi}r + A_1\frac{3}{4}r^2 \qquad (5)$$

Case I Cone

We can write $\eta_b(r)=r\tan\beta$ where β is the deadrise angle. It follows from Eq. (5) that $A_1=0$ and $A_0=\pi/4\tan\beta$. The solution of Eq. (4) is

$$c(t)=\frac{4}{\pi}\frac{Vt}{\tan\beta} = 1.27\frac{Vt}{\tan\beta} \qquad (6)$$

This does not agree with Chuang [15], who gets $c=1.36\,Vt/\tan\beta$. But the result agrees with Shiffman and Spencer [16]. The 2-D result for a wedge is $(\pi/2)Vt/\tan\beta$. This means that the rise of water is higher for a wedge than a cone with the same β at the same time. In calm water we have $Vt/\tan\beta$.

Case II Sphere (small submergences)

We can write $\eta_B=0.5\,r^2/R$ where R is radius of the sphere. It follows from Eq. (5) that $A_0=0$ and $A_1=2/(3R)$. Integration of Eq. (4) gives

$$c(t) = \sqrt{3RVt} \qquad (7)$$

for a sphere at small submergences. In calm water we have $\sqrt{2RVt}$. Wagner's method for a circular cylinder gives $2\sqrt{RVt}$. So our result is between a von Karman approach, i.e. no local rise of the water, and a Wagner approach for a circular cylinder. We note that dc/dt → ∞ when t → 0. This is similar as for a circular cylinder and indicates that compressibility of the water is important initially.

The ratio C_w between the vertical coordinate η_b corresponding to Eq. 7 and the submergence of the sphere with no local rise up of the water is time independent and equal to 1.5. This agrees with Ref. [16]. If the flow separates above η_b, C_w has a physical meaning as a wetting factor. This may not be so for a cone. A thin jet flow above the intersection line corresponding to Eq. (6) may stay attached to the cone. This case will be analyzed after the following matching.

Matching

We will match the inner 2-D jet flow solution by Wagner with our outer 3-D solution for axisymmetric flow. It follows from Eq. (1) that an inner expansion of the outer solution near r=c is

$$\phi \approx -\frac{2V}{\pi}\sqrt{2c}\sqrt{c-r} \qquad (8)$$

for r < c. The outer expansion of the inner expansion follows from earlier 2-D analysis in refs. [8] and [14] and is

$$-4\frac{dc}{dt}\sqrt{\frac{\delta}{\pi}}\sqrt{c-r} \qquad (9)$$

where δ is the jet thickness (see Fig. 3). Since Eq. (8) and (9) should be equal, it follows that

$$\delta = \frac{V^2}{2\pi}c\left(\frac{dc}{dt}\right)^{-2} \qquad (10)$$

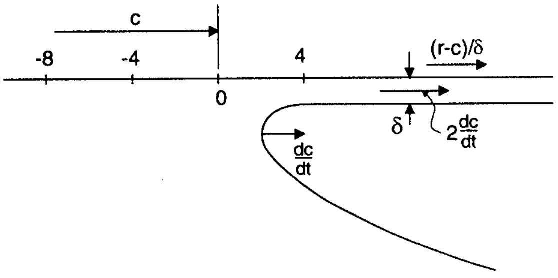

Fig. 3 2-D inner flow domain at the spray root during water entry of blunt axisymmetric body.

For a cone we get (see Eq. 6)

$$\delta_{cone} = \frac{1}{8}\tan\beta(Vt) \qquad (11)$$

The 2-D result for a wedge is twice the expression for a cone. For a sphere at small entrances we get

$$\delta_{sphere} = \frac{2}{\pi\sqrt{3}}\frac{(Vt)^{3/2}}{\sqrt{R}} \qquad (12)$$

Wagner's inner flow solution assumes that the jet flow has semi-infinite extent with thickness δ. In reality this is not so. Cointe [14] analyzed the details of the jet part of the flow for a 2-D wedge and matched with the inner flow by Wagner. Cointe assumed the jet stayed attached to the wedge. We will do similar for a cone. The detailed jet flow has a triangular form in the r-z plane with a length ℓVt and an intersection angle α between the cone surface and the free surface (see Fig. 4) The volume $Vol(t)$ of the detailed part of the jet flow around the cone is

$$Vol(t)=2\pi\alpha\{\frac{1}{2}(c+\ell Vt\cos\beta)(\ell Vt)^2 - \frac{1}{3}\cos\beta(\ell Vt)^3\}$$

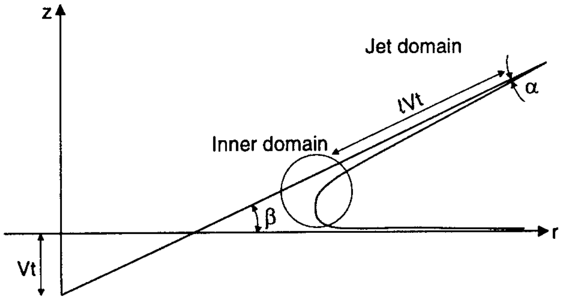

Fig. 4 Attached jet flow during water entry of blunt axisymmetric body.

It follows by using fluid mass continuity that $\rho dVol(t)/dt$ is equal to $\rho(dc/dt)\delta\, 2\pi c$, which is the fluid mass flux into the detailed part of the jet flow. This gives $\ell V=dc/dt$ and

$$\alpha=\frac{\delta}{\ell Vt}=\tan^2\beta\cdot\pi/32$$

The α-value for a wedge with the same β is 16 times as large. The wetting facta C_w for a cone with attached jet flow is then $4(1+1/\cos\beta)/\pi$ which for small angles β is approximately 2.5. A similar 2-D analysis for a wedge gives $C_w=\pi-1$.

Composite solution for the pressure

We will construct a solution for the pressure on the body that is valid both for the inner and outer flow. If only the outer flow description was used, it would lead to infinite pressures at r=c. The composite solution gives finite values for the pressure when t>0. We will construct the composite solution in the same way as Zhao and Faltinsen [8] did for the 2-D water entry problem. It follows from Eq. (1) and Bernoulli's equation that the leading order pressure in the outer domain on the body is

$$p_{out}-p_a=\frac{\rho 2V}{\pi}c\frac{dc}{dt}(c^2-r^2)^{-1/2}$$

p_a means atmospheric pressure and ρ is the mass density of water. The pressure in the inner domain follows from Eq. (4.2) in ref. [8] and is given by

$$p_{in}-p_a=2\rho[dc/dt]^2|\pi|^{1/2}(1+|\tau|^{1/2})^{-2} \qquad (13)$$

where $|\tau|$ is related to r by

$$r-c=(\delta/\pi)(-\ln|\tau|-4|\tau|^{1/2}-|\tau|+5) \qquad (14)$$

(see Eq. (4.3) in ref. [8]). δ is given by Eq. (10). The composite solution for the pressure on the body for $0 \le r \le c(t)$ is then (compare with Eq. (4.5) in ref. [8]).

$$p-p_a = \rho\frac{2V}{\pi}c\frac{dc}{dt}\left((c^2-r^2)^{-1/2}-(2c(c-r))^{-1/2}\right) + 2\rho[dc/dt]^2|\tau|^{1/2}(1+|\tau|^{1/2})^{-2} \qquad (15)$$

When $r > c(t)$, Eq. (14) is used. $c(t)$ is given by Eq. (6) for a cone and by Eq. (7) for a sphere. $|\tau|$ is related to r by Eq. (15) and δ is given by Eq. (11) for a cone and by Eq. (12) for a sphere.

The theory is an asymptotic theory valid for small local deadrise angles. The maximum pressure is $0.5\rho(dc/dt)^2+p_a$ and occurs at $r=c(t)$. The maximum pressure on a cone is then

$$(p_{\max})_{cone}-p_a=\rho\frac{8V^2}{\pi^2\tan^2\beta} \qquad (16)$$

Chuang predicts higher maximum value. The dominant pressure term in his expression for small β is $\rho 9.2V^2/\pi^2\beta^2$. This is consistent with that Chuang predicts higher values for dc/dt than us. The ratio between the maximum pressure on a cone and a wedge with the same value of β is with our analysis

$$\frac{(p_{\max})_{cone}-p_a}{(p_{\max})_{wedge}-p_a} = \frac{64}{\pi^2} = 0.66 \qquad (17)$$

Wagner's solution has been used for the wedge. Chuang [15] finds that this ratio is 0.75. This is based on Pierson's [19] analysis for a wedge. Takemoto [20] has compared Chuang's theory with Chuang and Milne's [21] experiments for a cone. He corrected for that the pressure gauge has a finite diameter and that the large pressures around maximum pressure have a small spacial extent. The latter becomes more and more pronounced the smaller β is. He presented comparisons for $\beta = 1^o, 3^o, 6^o, 10^o$ and 15^o. In all these cases the finite size of the pressure gauge represented an important correction. Further Takemoto corrected for the velocity of the cone is smaller when the pressure gauge comes into the water and maximum pressure occurs. Takemoto [20] documented good agreement with Chuang's theory except possibly for $\beta = 1^o$. Air cushion effects may then matter. Making similar corrections with our theory give also good agreement with the experiments. Actually the agreement is better except for $\beta = 6^o$ where it is difficult to say which theory agrees best. At the nose of the cone $(r=0)$ our analysis gives $(p_{nose})_{cone}-p_a = \rho 8V^2/(\pi^2\tan\beta)$. As long as $\beta < 45^o$, this is lower than $(p_{\max})_{cone}$. This result is consistent since $\beta = 45^o$ can hardly be called a small angle. The ratio between the pressure at the nose of a cone and a wedge is $[(p_{nose})_{cone}-p_a]/[p_{nose})_{wedge}-p_a] = 16/\pi^3 = 0.51$.

The maximum pressure on a sphere is time dependent while it is not for a cone with a constant water entry velocity. Actually when $t=0$ the pressure on a sphere is infinite based on the preceding analysis. Other physical effect like airflow and compressibility of the water will make the pressure finite at the initial time. Hydroelasticity will also matter. If we want to find the resulting local maximum structural stresses, this may be the most important part (Faltinsen [3]).

Vertical force

The vertical force F_3 can be obtained by properly integrating the composite solution for the pressure. An alternative approximate way is to use the outer solution. It follows that

$$F_3=\rho\frac{2V}{\pi}c\frac{dc}{dt}\int_0^c\frac{2\pi r dr}{\sqrt{c^2-r^2}} = \rho 4Vc^2\frac{dc}{dt} \qquad (18)$$

This is based on constant entry velocity V. If V changes, the added mass force $A_{33}dV/dt$ has to be added. By using Eq. (1) and the definition of added mass it follows that

$$A_{33}=\frac{2}{\pi}\rho\int_0^c\sqrt{c^2-r^2}\,2\pi r dr = \frac{4}{3}\rho c^3 \qquad (19)$$

The total water entry force can then be written

$$F_3 = \frac{d}{dt}(A_{33}V) \qquad (20)$$

Eq. [18] can be shown to be asymptotically correct to lowest order by using conservation of momentum as the fluid. The momentum flux into the jet flow is then higher order. It is also possible to show Eq. [18] by energy arguments. The hydrodynamic work done on the body is F_3V. This is equal to the sum of the flux of kinetic energy out into the jet and the rate of change with time of the kinetic energy in the fluid. The kinetic energy flux out into the jet is

$$\rho\frac{1}{2}(2\frac{dc}{dt})^2\delta 2\pi c\frac{dc}{dt}$$

By using Eq. (10) this can be written $\rho 2V^2c^2dc/dt$. The time derivative of the kinetic energy in the fluid is

$$\frac{d}{dt}(\frac{1}{2}A_{33}V^2) = 2\rho c^2\frac{dc}{dt}V^2$$

This follows by Eq. (19) and using that the kinetic energy can be related to the added mass. This proves Eq. (18) by energy arguments. Ref. [22] used similar arguments for 2-D flow.

By assuming V is constant and using Eq. (6) it follows that the force on a cone is

$$F_3 = 4(4/\pi)^3(Vt)^2\rho V^2/\tan^3\beta \qquad (21)$$

By using Eq. (7) it follows that the slamming force coefficient for a sphere at small submergences is

$$C_s = \frac{F_3}{0.5\rho\pi R^2 V^2} = \frac{12\sqrt{3}}{\pi}\left(\frac{Vt}{R}\right)^{1/2} \qquad (22)$$

A similar result has been derived in refs. [23] and [24]. Eq. (22) shows that the initial C_s-value is zero and increases with time. This is opposite of the behavior of the impact of horizontal circular cylinder which starts initially with a finite C_s-value with a Wagner type of analysis.

Miloh [25] has studied theoretically the impact of a sphere. His analysis did not incorporate the local rise up of the water. However, he satisfied the exact body boundary conditions for any submergence. Miloh [25] predicts a maximum value of $C_s=0.96$ at $Vt/R\approx 0.2$, which means a local deadrise angle β of 37^o at the intersection between the undisturbed free surface and the body. An asymptotic theory would be questionable at so large β-values. Miloh's analysis for small values of Vt/R could be compared with an asymptotic analysis with a von Karman approach. This give a slamming coefficient of $(8\sqrt{2}/\pi)$ $(Vt/R)^{1/2}$. Miloh [26] studied in details the asymptotic behavior for small values of Vt/R based on the solution in ref. [25]. Terms of $0((Vt/R)^{3/2})$ were included. He discussed the influence of wetting corrections. If he used $C_w= 1.5$ like in the asymptotic method presented in this chapter, he got

$$C_s = (12\sqrt{3}/\pi)\tau^{1/2} - 5.36\tau - 5.76\tau^{3/2} \qquad (23)$$

where $\tau = Vt/R$. The leading order term agrees with Eq. (22). He found by using his computational value $C_w=1.327$ that

$$C_s = 5.5\tau^{1/2} - 4.19\tau - 4.26\tau^{3/2} \qquad (24)$$

This agrees satisfactory with the experimental data in ref. [27] for $\sim 0.08 < \tau^{1/2} < \sim 0.53$. The maximum value of C_s based on Eq. (24) is 1.26 and occurs when $Vt/R=0.164$.

4. THREE-DIMENSIONAL NUMERICAL METHOD

The simplified 2-D water entry theory described in ref. [10] will be generalized to 3-D flow with an axisymmetric body. A coordinate system rz_1 which follows the vertical motion of the body is used (see Fig. 5). The problem is solved as an initial value problem. Green's second identity is used to represent the velocity potential ϕ as a distribution of Rankine sources and dipoles over the body surface and a horizontal plane outside the body that starts at the intersection line between the body and the free surface (see Fig. 5). The dynamic free surface condition $\phi=0$ is satisfied. ϕ is represented in the far-field as a vertical dipole. The dynamic free surface condition implies that fluid particles on the free surface have only vertical velocities. Fig. 5 illustrates the free surface l_i and l_{i+1} at two time instances t_i and t_{i+1}. The free-surface condition is satisfied on the horizontal planes L_i and L_{i+1} at t_i and t_{i+1}. An integral equation is used at each time step t_i to find the velocity potential on the body surface and the vertical velocities at L_i. It is assumed that the vertical velocity on L_i for a given r-value is the same as the vertical velocity on l_i. The solution can be stepped from $t = t_i$ to $t = t_{i+1}$ in the following way. The procedure is first to decide the intersection line (r_{i+1}, z_1^{i+1}) at time t_{i+1} and then find what $\Delta t = t_{i+1} - t_i$ must be. One can write

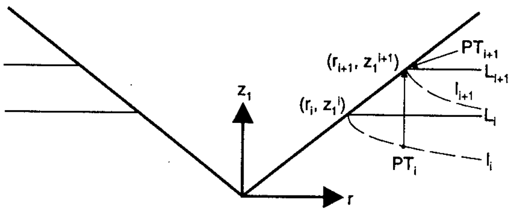

Fig. 5 The coordinate system (r, z_1), the real free surface ℓ and the horizontal lines L used in the numerical simulation for time steps t_i and t_{i+1} in the simplified solution of water entry of an axisymmetric body.

$$\Delta P = \int_{t_i}^{t_{i+1}} W(c(t), r_{i+1})dt \qquad (25)$$

where $\Delta P = PT_{i+1} - PT_i$ (see Fig. 5) and W is the relative vertical velocity of a point between PT_i and PT_{i+1} with r-coordinate r_{i+1}. W depends on c(t) which is the r-coordinate of the intersection line between the free surface and the body surface at time t. Eq. (25) can be written

$$\Delta P = \int_{r_i}^{r_{i+1}} W(c(t), r_{i+1})\frac{dt}{dc}dc \qquad (26)$$

This is evaluated numerically by using an average value $(dt/dc)_m$ for dt/dc. W is expressed in terms of a local solution. Here a local polar coordinate system (R_2, θ_2) is used (see Fig. 6). W can be written as

$$W = -(D/(2-\gamma/\pi))R_2^{(2-\gamma/\pi)^{-1}-1} \qquad (27)$$

where R_2 is $(r_{i+1}-c(t))$. γ is defined in Fig. 6 and is assumed constant from t_i to t_{i+1}. D is found from the global solution. An average value D_m for the two time instances t_i and t_{i+1} is used. Eq. (26) can now be analytically integrated. It follows that

$$\left(\frac{dt}{dc}\right)_m = -\frac{\Delta P}{D_m(r_{i+1}-r_i)^{(2-\gamma/\pi)^{-1}}} \qquad (28)$$

Δt is now determined as $(dt/dc)_m(r_{i+1}-r_i)$.

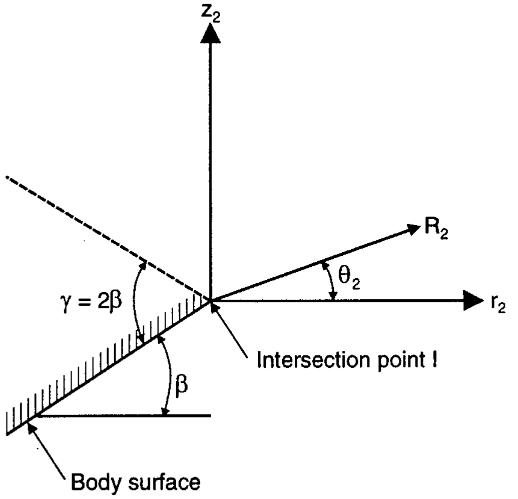

Fig. 6 Local polar coordinate system (R_2, θ_2) and local Cartesian coordinate system (r_2, z_2). I is intersection between the body and the free surface β=local deadrise angle. Free surface condition is satisfied on the r_2-axis.

After the velocity potential on the body surface has been determined, the pressure distribution on the body can be found from Bernoulli's equation. Special care is shown near the intersection between the body and the free surface. Since the velocity is infinite there, the velocity square term in Bernoulli's equation will be negative infinite. It can be shown that the $-\rho\partial\phi/\partial t$-term is positive infinite at the intersection point, and that the velocity square term is more singular than the $-\rho\partial\phi/\partial t$-term. Therefore the total pressure is negative infinite. But this is an integrable singularity. Let us define the integrated vertical force for the part with negative pressure as F_N, and from the part with positive pressure as F_P. It can be shown that F_N/F_P goes to zero when the deadrise angle goes to zero. For small deadrise angles, the maximum pressure (positive) is obtained near the intersection point.

4.1 Verification and validation

It was documented in chapter 3 that maximum pressures predicted by the asymptotic method on cones with deadrise angles β between 3^o and 15^o are in good agreement with experimental values. Fig. 7 shows the pressure distribution on a cone with $\beta = 10^o$. The agreement between the numerical method and the asymptotic method given by Eq. (15) is good. Similar comparisons were made for β between 5^o and 30^o. The smaller β is, the better the agreement is. Results for $\beta = 30^o$ are shown in Fig. 8. The asymptotic method shows a small nearly non-noticeable kink in the pressure distribution around maximum pressure. This is an indication that β is too large for the asymptotic theory to be

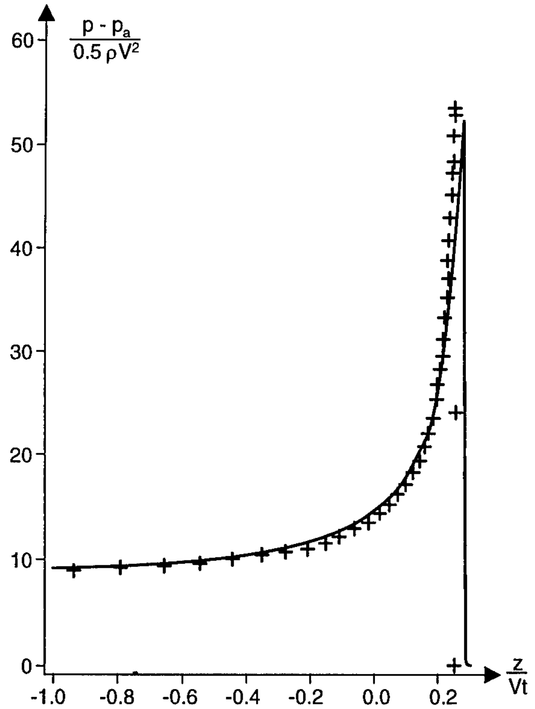

Fig. 7 Prediction of the pressure distribution during water entry of a cone with constant downwards velocity V. Deadrise angle $\beta = 10^o$, ++++, numerical method; ——, asymptotic solution.

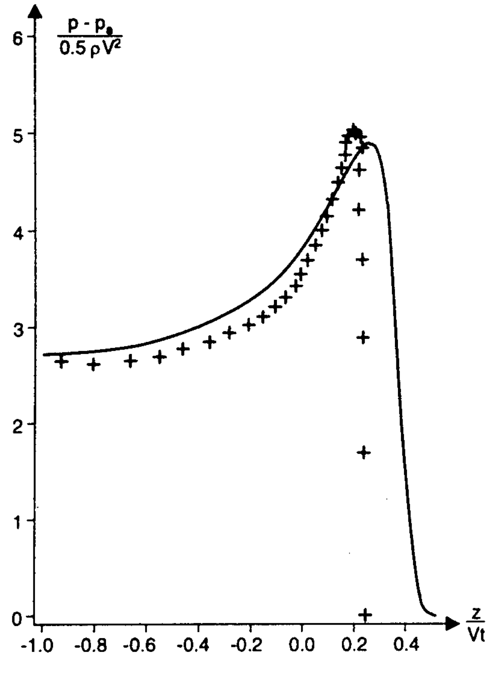

Fig. 8 Predictions of the pressure distribution during water entry of a cone with constant downwards velocity V, Deadrise angle $\beta = 30^o$. ++++, numerical method; ——, asymptotic solution.

valid. We note that the magnitudes of the predicted pressures are similar, but the numerical method predicts less rise up of the water than the asymptotic method. This is also evident from Fig. 9 where the wetting factor $C_w=\eta_b/(Vt)$ (see Fig. 1) is presented versus β. The vertical force as a function of

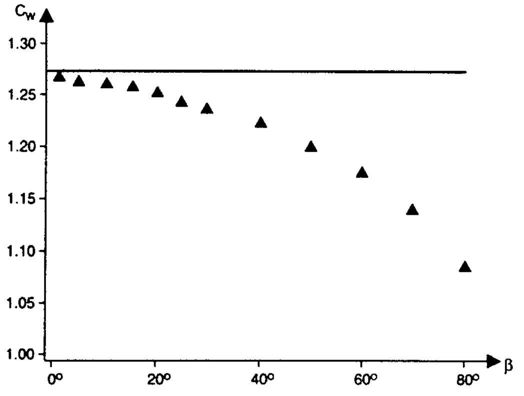

Fig. 9 Wetting factor C_w during water entry of a cone with constant downwards velocity. Presented as a function of deadrise angle β. ▲▲▲▲, numerical method; ——, asymptotic solution.

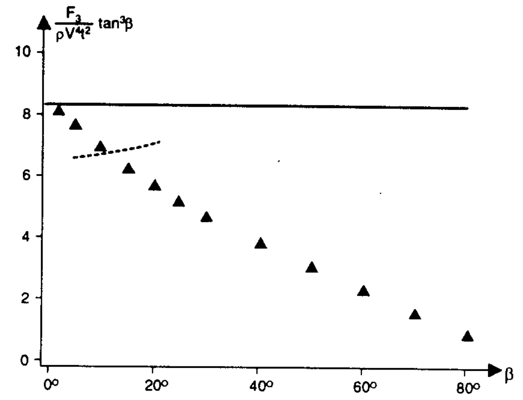

Fig. 10 Non-dimensionalized vertical force F_3 during water entry of a cone with constant downwards velocity V. Presented as a function of deadrise angle β. ▲▲▲▲, numerical method; ——, asymptotic solution; ----- experiments (Watanabe [28]).

β is shown in Fig. 10. The force calculated by the numerical method is obtained by pressure integration, while the asymptotic method is based on Eq. (21). The force has been non-dimensionalized so that the asymptotic method predicts a constant non-dimensionalized value as a function β. The numerical method predicts a decreasing non-dimensionalized force with increasing β-value. This is consistent with the wetting factor in Fig. 9 and that the two different methods predict similar magnitudes of pressures for $\beta\leq 30^o$. The two methods agree when $\beta \to 0$. Experimental values by Watanabe [28] are also presented in Fig. 10. Watanabe performed drop tests for β between 5^o and 20^o. An empirical force formula that depends on the structural mass M of the dropped object was presented. The results in Fig. 10 are obtained by letting $M \to \infty$. The agreement between the numerical method and the experiments is good except that the experimental non-dimensional values show a small increase with increasing β.

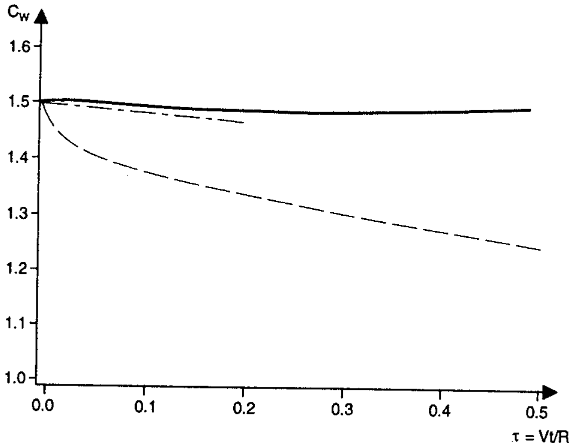

Fig. 11 Wetting factor C_w during water entry of a sphere with constant downwards velocity V versus dimensionless time. ———, numerical method; - - - - - - - - -, experiments (Cooper [29]);— — —, experiments (Shiffman and Spencer [23]).

Fig. 11 shows the wetting factor as a function of $\tau=Vt/R$ during water entry of a sphere with constant downwards velocity V. Experimental results by Cooper [29] for $0<\tau<0.2$ and by Shiffman and Spencer [23] for $0.2<\tau<0.8$ are presented together with results by the numerical method. The asymptotic method predicts $C_w=1.5$ which is in good agreement with the numerical method and the experimental results in ref. [29]. The agreement with ref. [23] is less good.

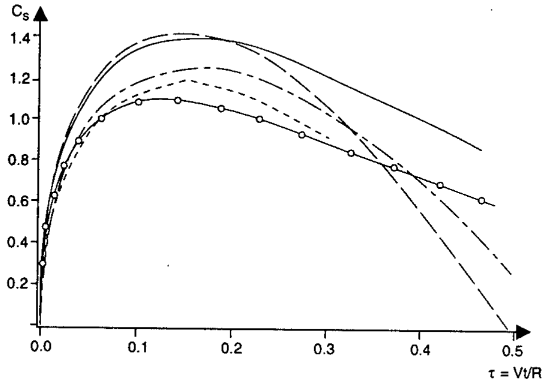

Fig. 12 Slamming coefficient C_s during water entry of sphere with constant downwards velocity V versus dimensionless time.– ○ – ○–, numerical method and pressure integration; —— numerical method and VdA_{33}/dt; ———, Eq. (23) (Miloh [26]); — — — —, Eq. (24) (Miloh [26]); - - - - - - - -, experiments (Moghisi and Squire [27]).

Fig. 12 shows the slamming force coefficient C_s defined by Eq. (22) as a function of τ. Eqs. [23] and [24] proposed by Miloh (26) are plotted together with results by the numerical method and the experimental results by Moghisi and Squire [27]. The force calculated by the numerical method is obtained by pressure integration. The quantity VdA_{33}/dt where A_{33} is our calculated infinite frequency added mass in heave as a function of submergence, is also plotted in Fig. 12. When $\tau \rightarrow 0$ (see the discussion in chapter 3), VdA_{33}/dt is the vertical force based on conservation of momentum in the fluid. The derivation requires that the pressure is atmospheric on the free surface. When τ is finite, the condition of atmospheric pressure on the free surface is not satisfactorily satisfied with the "ϕ=0 condition" on the free surface. The quadratic velocity term in the Bernoulli's equation for the pressure cannot be neglected. This is why VdA_{33}/dt for finite τ does not agree with the force obtained by pressure integration. The experimental results for $\tau<0.16$ are based on the empirical relation found by regression analysis in ref. (27). The results for $0.16<\tau<0.3$ are based on own judgement of the experimental results. We note that both Eq. (24) and the numerical method based on pressure integration agree quite well with the experimental results. When $\tau \rightarrow 0$, the numerical method agrees with the asymptotic results given by Eqs. (22) and (23), but not with Eq. (24). When $\tau > \sim 0.25$, the numerical method and Eqs (23) and (24) have a different trend. The reason is that Eqs. (23) and (24) are based on τ being small. Eq. (23) agrees quite well with our calculations of VdA_{33}/dt up to $\tau \sim 0.25$. The reason is that Miloh's derivation of Eq. (23) is consistent with using VdA_{33}/dt as the vertical force together with a wetting factor of 1.5. Our results show that it is better to use pressure integration than calculating the force as VdA_{33}/dt.

These verification and validation tests document that the numerical method is a valuable tool for studying water entry of three-dimensional bodies. However, since the atmospheric pressure condition is not satisfactorily satisfied on the free surface for finite values of the local deadrise angles, it is recommended in the future to generalize the more complete 2-D method presented in ref. [8] to 3-D flow.

5. CONCLUSIONS

Theoretical methods for water entry of two-dimensional and axisymmetric bodies are discussed. When the local angle between the water surface and the body surface is very small, hydroelasticity should be considered. The presented analysis of water entry loads assumes that the local angle between the water surface and the free surface is not very small. The structure can be considered rigid and compressibility effects can be neglected. Two 2-D numerical methods are discussed. Both methods satisfy the exact body boundary condition and account for the local rise up of the water. One of the methods satisfies the exact free surface condition without gravity. The studies show that it is more appropriate to approximate the free surface conditions than the body boundary condition. Flow separation from sharp corners has been incorporated. Gravity has been neglected, but should be incorporated after flow separation occurs. The loads after flow separation have been found to be non-negligible. The simplified method has been generalized to 3-D axisymmetric flow without flow separation. In order to verify the 3-D method, water entry of axisymmetric bodies with small local deadrise angles are studied analytically. An inner domain solution for the jet flow is matched to an outer solution and a composite solution for the pressure is presented. Verification and validation tests for spheres and cones are reported. It is demonstrated that the local rise-up of the water is also important during water entry of a blunt 3-D structure. A common formula for the impact force in terms of the time derivative of added mass is discussed. It is only recommended to use this formula as long as the local deadrise angle is small.

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